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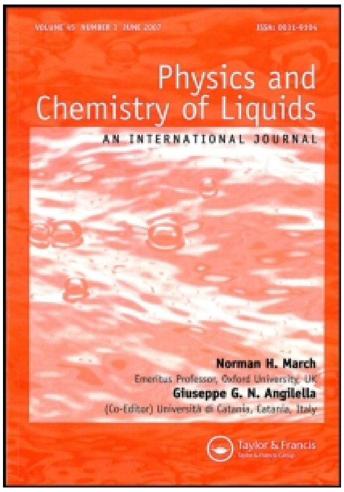

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On the Hydrodynamics of a Superfluid With a Three-particle "Condensate"

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Since 1974 there have been serious experimental and also theoretical pointers to the possibility that no Bose condensation occurs in superfluid He^4. Evans presented a theoretical argument against a condensate, suggesting a "pairing" model of superfluid helium. As yet, experiment cannot decide if the concept of a one-particle condensate can be replaced by the idea of a two-particle condensate. March and Galasiewicz have argued that, if there is no one-particle condensate in He^4, a ground state wave function cannot be built from a product of pairs but must fundamentally include three-particle correlations at least. So the idea of a three (not two) particle condensate seems worth some attention. It is demonstrated here that assuming a three-particle condensate we can regain from microscopic theory the Landau hydrodynamic equation for the superfluid velocity.

1 INTRODUCTION

In 1966 Hohenberg and Platzman[1] proposed an experiment to verify the existence of a Bose condensate in superfluid He^4. The experiment is based on inelastic neutron scattering measurements at large momentum transfers.

A number of experiments subsequently performed suggested a non-zero fraction of Bose condensate (named later one-particle condensate) in the total density of superfluid He^4. For example, Cowley and Woods[2] estimated the condensate fraction to be 17%. The experiment by Mook, Scherm and Wilkinson[3] gave 2.4% condensate. In 1974, Jackson[4] re-examined the data of paper[3] and found that if a condensate were present the fraction would have to be less than 1%. In 1975 experiments by Cowley[5] and coworkers raised serious doubts on the existence of the condensate. In the same year Evans[6] presented a theoretical argument against a condensate in superfluid He^4. At the end of his paper, Evans suggested a "pairing" model of superfluid helium as a Boson analogue of the theory of superconductivity. Many versions of such a model have been proposed since the formulation of the theory of superconductivity.

One can formally propose two kinds of experimental tests, connected with the a.c. Josephson effect and values of circulation quanta, which could decide between the ideas of one or two-particle condensate.

Thus Josephson's a.c. component of frequency would be expected to be $n\Delta\mu/\hbar$, $n = 1, 2$, in the case of n-particle condensate ($\Delta\mu$ denotes a change of chemical potential across a link). The first attempt to observe the a.c. Josephson effect was made by Richards and Anderson[7] and later by others.[8,9] All these experiments have been criticized by Musinski.[10] So it seems that a completely convincing experiment of this kind has still to be performed and it does not yet seem possible to say anything decisive about the numerical coefficient in the frequency of the Josephson current.

The second test is connected with the problem of the circulation quantum in superfluid He^4. For the one-particle condensate, the theoretical prediction is that the velocity circulation quantum is equal to h/m. This value has been confirmed experimentally.[11,12] For the two particle condensate the theoretical situation is confused. Thus in paper[13] it is argued that the quantum is $h/2m$, while in paper[6] it is claimed to be h/m.

It seems that, at present, experiment cannot decide whether the concept of a one-particle condensate can be replaced by the idea of a two-particle condensate. Suggestions concerning this problem follow from the theoretical work of March and Galasiewicz.[14] It is argued there that if there is no (one-particle) condensate in He^4, then the He–He interactions must lead to a ground state wave function which cannot be built, like Bijl–Jastrow wave function, from a product of pairs. It must include fundamentally three-atom correlations (at least). This suggests that if experiments eventually exclude a one-particle condensate in He^4, it would be of interest to consider the idea of three (not two)-particle condensate. In such a situation, it would, of course, be important to have experimental evidence on three-atom correlations in He^4. Indeed, a method has been proposed to investigate such three-particle correlations in liquids[15] by studying the pressure dependence of the structure factor $S(k)$. This method has been used in classical liquids. An investigation of triplet correlations in He^4 has been performed in paper.[17] Experiment shows the existence of these correlations but the uncertainty in the experiment is $\pm 30\%$. It seems that the method in Ref. 15 could give a more precise answer to the question raised here because by studying triplet correlations via pair correlations one can omit contribution from three body forces.

The aim of this paper is to demonstrate that assuming a three-particle condensate an important and well established result in the theory of super-

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fluid He^4, namely the Landau hydrodynamic equation for the superfluid velocity, can be recovered. Use will be made of the microscopic approach of Bogoliubov.[17]

2 HYDRODYNAMIC EQUATION FOR SUPERFLUID VELOCITY

In order to get hydrodynamic equations we must derive the continuity equations of the form

$$\frac{\partial a}{\partial t} + \operatorname{div} j^{a} = f[\eta, \eta^{*}, U] \tag{2.1}$$

for the particle, current and energy density and an extra equation giving time derivative for superfluid velocity. Term on the right hand side of (2.1) depends on some external fields. We can derive equations of type (2.1) from microscopic theory[17,18] after calculation of commutators of operators representing suitable densities with total Hamiltonian. These commutators we then average with the density matrix. The form of equations is in evident way independent on our assumption that one-particle condensate is not present and we have strong three-particle correlations leading to three-particle "condensate." Our assumption can be written down

$$\begin{gathered} \langle \psi(t, r) \rangle = 0, \qquad \langle \psi(t, r_1)\psi(t, r_2) \rangle = 0, \\ \langle \psi(t, r_1)\psi(t, r_2)\psi(t, r_3) \rangle \neq 0 \end{gathered} \tag{2.2}$$

where ψ denote Bose field operators and $\langle \ldots . \rangle$ averaging with density matrix.

Consider a system of interacting bosons described by Hamiltonian

$$\hat{H} = \hat{H}' - \lambda \hat{N} + \hat{H}_t, \tag{2.3}$$

$$\begin{aligned} \hat{H}' = {} & \frac{\hbar^2}{2m} \int \nabla\psi^{+}(t, r)\nabla\psi(t, r)\mathrm{d}r \\ & + \frac{1}{2} \iint V(r - r')\psi^{+}(t, r)\psi^{+}(t, r')\psi(t, r')\psi(t, r)\mathrm{d}r\, \mathrm{d}r', \end{aligned} \tag{2.4}$$

$$\hat{N} = \int \psi^{+}(t, r)\psi(t, r)\mathrm{d}r = \int \hat{\rho}(t, r)\mathrm{d}r, \tag{2.5}$$

$$\begin{aligned} \hat{H}_t = {} & \int U(t, r)\hat{\rho}(t, r)\mathrm{d}r \\ & + \frac{1}{3} \iiint \{\eta(t \mid r_1, r_2, r_3)\hat{\Phi}_3^{+}(t \mid r_1, r_2, r_3) \\ & + \eta^{*}(t \mid r_1, r_2, r_3)\hat{\Phi}_3(t \mid r_1, r_2, r_3)\mathrm{d}r_1\, \mathrm{d}r_2\, \mathrm{d}r_3\} \end{aligned} \tag{2.6}$$

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where

$$\hat{\Phi}_3(t|r_1, r_2, r_3) = \psi(t, r_1)\psi(t, r_2)\psi(t, r_3),$$
$$\eta(t|r_1, r_2, r_3) = \eta(t|r_2, r_1, r_2) = \eta(t|r_3, r_2, r_1) = \cdots \tag{2.7}$$

In (2.5) λ is a constant playing a role of chemical potential, U in (2.6) denotes external, time dependent scalar potential and η, η^* are generalization of "sources of particles" introduced in Ref. 17. Because of assumption (2.2) it was possible to introduce terms with η, η^* into (2.3).

In order to find time derivative of $\hat{\Phi}_3$ it is convenient to have

$$i\hbar \frac{\partial\psi(t, r)}{\partial t} = [\psi, \hat{H}] = \left\{-\frac{\hbar^2}{2m}\nabla^2 - \lambda + U + \int V(r - r')\hat{\rho}(t, r')\mathrm{d}r'\right\}\psi(t, r)$$
$$+ \iint \eta(t|r, r', r'')\psi^+(t, r'')\psi^+(t, r')\mathrm{d}r'\,\mathrm{d}r''. \tag{2.8}$$

For $\hat{\Phi}_3$ we find

$$i\hbar \frac{\partial}{\partial t}\hat{\Phi}_3(t|r_1, r_2, r_3) = [\hat{\Phi}_3, \hat{H}]$$
$$= \left\{\sum_{i/1}^{3}\left[-\frac{\hbar^2}{2m}\nabla_i^2 - \lambda + U(t, r_i) + \int V(r' - r_i)\hat{\rho}(t, r')\mathrm{d}r'\right]\right.$$
$$\left. + \frac{1}{3}[V(r_1 - r_2) + V(r_2 - r_3) + V(r_3 - r_1)]\right\}$$
$$\times \hat{\Phi}_3(t|r_1, r_2, r_3) + \hat{F}[\eta] \tag{2.9}$$

where

$$\hat{F}[\eta] = 2\eta(t|r_1, r_2, r_3)$$
$$+ \sum_{i,j,l/1}^{3} e_{ijl}\left[\iint \eta(t|r_i, r', r'')\mathrm{d}r'\,\mathrm{d}r''\psi^+(t, r')\psi^+(t, r'')\psi(t, r_j)\psi(t, r_l)\right.$$
$$\left. + 2\int \eta(t|r_i, r_j, r')\mathrm{d}r'\psi^+(t, r')\psi(t, r_l)\right] \tag{2.10}$$

and

$$\begin{aligned} e_{ijl} &= 1 \quad \text{for} \quad i \neq j \neq l, i \neq l, \\ e_{ijl} &= 0 \quad \text{for} \quad i = j \text{ or } i = l \text{ or } j = l. \end{aligned} \tag{2.11}$$

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We perform averaging of (2.9) with density matrix

$$i\hbar\frac{\partial}{\partial t}\langle\hat{\Phi}_3\rangle = \sum_{i/1}^{3}\left\{-\frac{\hbar^2}{2m}\nabla_i^2 - \lambda + U(t, r_i)\right.$$
$$\left. + \frac{1}{3}[V(r_1 - r_2) + V(r_2 - r_3) + V(r_3 - r_1)]\right\}\langle\hat{\Phi}_3\rangle$$
$$+ \sum_{i/1}^{3}\int V(r' - r_i)\mathrm{d}r'\langle\hat{\rho}(t, r')\hat{\Phi}_3\rangle + \langle\hat{F}[\eta]\rangle. \quad (2.12)$$

We introduce now e.g. Jacobi coordinates (r, R_1, R_2)

$$r = \tfrac{1}{3}(r_1 + r_2 + r_3), R_1 = r_1 - r_2, R_2 = \tfrac{1}{2}(r_1 + r_2) - r_3,$$
$$r_1 = r + \tfrac{1}{2}R_1 + \tfrac{1}{3}R_2, r_2 = r - \tfrac{1}{2}R_1 + \tfrac{1}{3}R_2, r_3 = r - \tfrac{2}{3}R_2, \quad (2.13)$$
$$R_i = (X_i, Y_i, Z_i).$$

In new coordinates we have

$$\sum_{i/1}^{3}\nabla_i^2 = \frac{1}{3}\nabla^2 + 2\frac{\partial^2}{\partial R_1^2} + \frac{3}{2}\frac{\partial^2}{\partial R_2^2}, \nabla^2 = \frac{\partial^2}{\partial r^2}. \quad (2.14)$$

We denote

$$\Phi_3(t|r, R_1, R_2) = \langle\hat{\Phi}_3(t|r_1, r_2, r_3)\rangle \quad (2.15)$$

and perform averaging of (2.12) with respect of R_1 and R_2 i.e. integration $1/V_0^2\iint\cdots\mathrm{d}R_1\,\mathrm{d}R_2$. Volume $V_0 \sim \xi_0^3$ where ξ_0 is a correlation length connected with triplets of particles. For distances between particles greater than ξ_0 $\langle\psi(t, r_1)\psi(t, r_2)\psi(t, r_3)\rangle = 0$ i.e. outside V_0 correlation function $\langle\hat{\Phi}_3\rangle = 0$.

We average correlation function

$$\Phi(t, r) = \overline{\langle\hat{\Phi}_3(t|r, R_1, R_2)\rangle} = \frac{1}{V_0^2}\iint\Phi_3(t|r, R_1, R_2)\mathrm{d}R_1\,\mathrm{d}R_2 \quad (2.16)$$

Because

$$\frac{1}{V_0}\int\frac{\partial^2}{\partial R_1^2}\Phi_3\,\mathrm{d}R_1 = \frac{1}{V_0}\sum_{j/1}^{3}\int_{V_0}\frac{\partial^2}{\partial X_{1j}^2}\Phi_3\,\mathrm{d}X_1\,\mathrm{d}Y_1\,\mathrm{d}Z_1 = 0 \quad (2.17)$$

(2.14) gives

$$\sum_{i/1}^{3}\nabla_i^2\Phi = \frac{1}{3}\nabla^2\Phi. \quad (2.18)$$

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We assume that U is a function varying very slowly in space. We denote

$$Z(t, r) = \frac{1}{V_0^2} \iint \left\{ \frac{1}{3} [V(R_1) + V(R_2 - \tfrac{1}{2}R_1) + V(R_2 + \tfrac{1}{2}R_1)] \langle \hat{\Phi}_3 \rangle + \sum_{i/1}^{3} \int V(r' - r_i) dr' \langle \hat{\rho}(t, r') \hat{\Phi}_3 \rangle \right\} dR_1 \, dR_2,$$

$$\bar{F}[\eta] = \frac{1}{V_0^2} \iint \langle \hat{F}[\eta] \rangle dR_1 \, dR_2, \tag{2.19}$$

$$\Phi(t, r) = \sqrt{\rho_{03}}\, a(t, r) e^{i\chi(t, r)} \tag{2.20}$$

where ρ_{03} denotes density of condensate at equilibrium.

Now (2.12) gives

$$\begin{aligned} i\hbar \frac{\partial \Phi}{\partial t} &= \left[-\frac{\hbar^2}{6m} \nabla^2 - 3\lambda + U(t, r) \right] \Phi(t, r) \\ &\quad + Z(t, r) + \bar{F}[\eta], \\ i\hbar \frac{\partial \Phi^*}{\partial t} &= \left[\frac{\hbar^2}{6m} \nabla^2 - 3\lambda - U(t, r) \right] \Phi^*(t, r) \\ &\quad - Z^*(t, r) - \bar{F}^*[\eta] \end{aligned} \tag{2.21}$$

From (2.20) we have

$$\begin{aligned} \hbar \frac{\partial \chi}{\partial t} &= \frac{1}{2\rho_{03} a^2} \left[\Phi i\hbar \frac{\partial \Phi^*}{\partial t} - \Phi^* i\hbar \frac{\partial \Phi}{\partial t} \right], \\ \rho_{03} \frac{\partial a^2}{\partial t} &= -\frac{1}{\hbar} \left[\Phi i\hbar \frac{\partial \Phi^*}{\partial t} + \Phi^* i\hbar \frac{\partial \Phi}{\partial t} \right] \end{aligned} \tag{2.22}$$

which allows us to use (2.21).

Equation for χ reads

$$\hbar \frac{\partial \chi}{\partial t} = 3\lambda + \frac{\hbar^2}{6m} \frac{\nabla^2 a}{a} - \frac{1}{2} \left(\frac{\hbar \nabla \chi}{3m} \right)^2 - \frac{1}{\rho_{03} a^2} (Z^* \Phi + Z \Phi^*) - U - \frac{e^{i\chi} \bar{F}^*[\eta] + e^{-i\chi} \bar{F}[\eta]}{2a\sqrt{\rho_{03}}}. \tag{2.23}$$

We define now the velocity of a superfluid component

$$v_s = \frac{\hbar}{M} \nabla \chi, \qquad M = 3m \tag{2.24}$$

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and denote

$$X = Z\Phi^* = \frac{1}{V_0^2} \iint \left\{ \frac{1}{3} \left[V(R_1) + V(R_2 - \tfrac{1}{2}R_1) + V(R_2 + \tfrac{1}{2}R_1) \right] \times \langle \hat{\Phi}_3 \rangle \overline{\langle \hat{\Phi}_3^+ \rangle} + \sum_{i/1}^{3} \int V(r' - r_i) dr' \langle \hat{\rho}(t, r') \hat{\Phi}_3 \rangle \overline{\langle \hat{\Phi}_3^+ \rangle} \right\} dR_1 \, dR_2 \tag{2.25}$$

Taking the gradient of both sides of (2.23) and using the definition (2.24) we get

$$M \frac{\partial v_s}{\partial t} = \nabla \left[\frac{\hbar^2 \nabla^2}{2Ma} - \frac{1}{2} M v_s^2 - \frac{1}{\rho_{03} a^2} (X + X^*) \right] - \nabla U - \nabla \frac{e^{i\chi} \tilde{F}^*[\eta] + e^{-i\chi} \tilde{F}[\eta]}{2a\sqrt{\rho_{03}}} \tag{2.26}$$

After averaging of equation (2.9) with the density matrix we describe our system in terms of parameters: particle density ρ, temperature θ, components of superfluid velocity v_s and velocity of normal component v_n. For proper introduction of v_n see Ref. 18. At the beginning we consider a system with one velocity v_s. At thermodynamic equilibrium $U = 0$, $\eta = \eta^* = 0$ all averages are functions of ρ, θ, $u = v_s^2/2$, which are constants in space and time, e.g.

$$\lambda \to \Lambda(\rho, \theta, u). \tag{2.27}$$

Equation (2.23) gives for equilibrium

$$\left[-\frac{\hbar^2}{4M} \frac{\nabla^2 a}{a} + \frac{1}{2} M v_s^2 + \frac{1}{\rho_{03} a^2} (X + X^*) \right]_{eq} = 3\Lambda(\rho, \theta, u). \tag{2.28}$$

For nonequilibrium where our parameters are local quantities

$$\rho = \rho(t, r), \qquad \theta = \theta(t, r), \qquad v_s = v_s(t, r)$$

we have

$$\left[-\frac{\hbar^2}{4M} \frac{\nabla^2 a(\rho, \theta, u)}{a(\rho, \theta, u)} + \frac{1}{2} M v_s^2(t, r) + \frac{1}{\rho_{03} a^2(\rho, \theta, u)} \times (X(\rho, \theta, v_s) + X^*(\rho, \theta, v_s)) \right]_{n, eq} = 3\Lambda(\rho, \theta, u) + \Lambda^1 \tag{2.29}$$

here Λ^1 contains dissipative terms which we will neglect. In (2.23) λ is still a constant i.e. $\nabla\lambda = 0$.

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The averages in (2.12) depend generally on ρ, θ, v_s, v_n but it is convenient to design them only by velocities i.e.

$$\langle\ \rangle_{v_s, v_n} \tag{2.30}$$

They refer to laboratory reference system and can be expressed in terms of averages taken in reference system mooving with normal component (in this system $v_n = 0$) i.e. in terms of

$$\langle\ \rangle_{v_s - v_n, 0} \tag{2.31}$$

We can change from laboratory reference system to reference system with $v_n = 0$ performing Galilean transformation for Bose amplitudes[17,18]

$$\psi(t, r) \to \psi(t, r)\exp\frac{imrv_n}{\hbar} \tag{2.32}$$

In this case (2.29) gives

$$\frac{1}{\rho_{03}a^2(\rho, \theta, u)}[X(\rho, \theta, v_s - v_n) + X^*(\rho, \theta, v_s - v_n)]$$
$$= \frac{\hbar^2}{4M}\frac{\nabla^2 a(\rho, \theta, u)}{a(\rho, \theta, u)} - \frac{1}{2}M(v_s - v_n)^2 + \Lambda(\rho, \theta, u), \qquad u = \tfrac{1}{2}(v_s - v_n)^2. \tag{2.33}$$

From (2.25) we see that $X(v_s, v_n) = X(v_s - v_n)$ because X is not affected by transformation (2.32).

With help of (2.33) equation (2.26) has a form

$$M\frac{\partial v_s}{\partial t} = -\nabla\left[\frac{Mv_s^2}{2} - \frac{M}{2}(v_s - v_n)^2 + 3\Lambda\right]$$
$$- \nabla U - \nabla\frac{e^{i\chi}\tilde{F}^*[\eta] + e^{-i\chi}\tilde{F}[\eta]}{2a\sqrt{\rho_{03}}} \tag{2.34}$$

Equation (2.34) is precisely the Landau hydrodynamic equation for superfluid helium, giving change in time of the superfluid velocity.

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